

AP Calculus AB

Approx Area Using Riemann Sums - Table

1)

$$\begin{aligned}
 a) T_4 &= \frac{1}{2} [f(0) + f(0.25)] \cdot (0.25) + \frac{1}{2} [f(0.25) + f(0.5)] \cdot (0.25) + \frac{1}{2} [f(0.5) + f(0.75)] \cdot (0.25) \\
 &\quad + \frac{1}{2} [f(0.75) + f(1.0)] \cdot (0.25) \\
 &= \frac{1}{2} \left(\frac{1}{4} \right) [1.0 + 2(0.8) + 2(1.3) + 2(1.1) + 1.6]
 \end{aligned}$$

$$\begin{aligned}
 b) R_4 &= \frac{1}{4} [f(1.0) + f(0.75) + f(0.5) + f(0.25)] \\
 &= \frac{1}{4} [1.6 + 1.1 + 1.3 + 0.8]
 \end{aligned}$$

$$\begin{aligned}
 c) M_2 &= f(0.25) \cdot (0.5) + f(0.75) \cdot (0.5) \\
 &= 0.5 [0.8 + 1.1]
 \end{aligned}$$

$$\begin{aligned}
 2) a) T_{10} &= \frac{1}{2} [f(0) + f(4)] \cdot 4 + \frac{1}{2} [f(4) + f(16)] \cdot 12 + \frac{1}{2} [f(16) + f(17)] \cdot 1 + \frac{1}{2} [f(17) + f(20)] \cdot 3 \\
 &= 2[8+4] + 6[4+6] + \frac{1}{2}[6+3] + \frac{3}{2}[3+6]
 \end{aligned}$$

$$\begin{aligned}
 b) R_{10} &= f(20) \cdot 3 + f(17) \cdot 1 + f(16) \cdot 12 + f(4) \cdot 4 \\
 &= (6)(3) + (3) + (6)(12) + (4)(4)
 \end{aligned}$$

3)

$$\begin{aligned}
 a) L_5 &= f(0) \cdot 2 + f(2) \cdot 2 + f(4) \cdot 2 + f(6) \cdot 2 + f(8) \cdot 10 \\
 &= 2[12.5 + 13.4 + 13.9 + 14.3 + 14.6]
 \end{aligned}$$

$$b) f'(5) \approx \frac{f(6) - f(4)}{6-4} = \frac{14.3 - 13.9}{2}$$

$$c) ROC = \frac{f(10) - f(0)}{10-0} = \frac{14.8 - 12.5}{10}$$

$$\begin{aligned}
 d) \int_0^{10} f'(x) dx &= [f(x) + C] \Big|_0^{10} \\
 &= f(10) - f(0) \\
 &= 14.8 - 12.5
 \end{aligned}$$

4)

$$\begin{aligned}
 a) T_4 &= \frac{1}{2} [R(0) + R(3)] \cdot 3 + \frac{1}{2} [R(3) + R(5)] \cdot 2 + \frac{1}{2} [R(5) + R(7)] \cdot 4 + \frac{1}{2} [R(7) + R(11)] \cdot 2 \\
 &= \frac{3}{2} [20 + 18] + [18 + 12] + 2 [12 + 15] + [15 + 19]
 \end{aligned}$$

$$b) R'(4) \approx \frac{R(5) - R(3)}{5 - 3} = \frac{12 - 18}{2}$$

$$\begin{aligned}
 c) ROC &= \frac{R(11) - R(0)}{11 - 0} \\
 &= \frac{19 - 20}{11}
 \end{aligned}$$

$$d) \int_0^{11} (3 + R'(t)) dt$$

$$\begin{aligned}
 &\left[3t + R(t) + C \right] \Big|_0^{11} \\
 &[33 + R(11)] - [0 + R(0)] \\
 &[33 + 19] - [20]
 \end{aligned}$$

